Mobility of Hot Carriers in Germanium at 300 °K

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The mobility of hot electrons and holes in germanium at a lattice temperature of 300 °K is calculated as dependent on carrier temperature and electric field intensity including not only the acoustical and nonpolar optical mode scattering but also the ionized impurity scattering. The Conwell theory of lattice mobility of hot carriers and the Conwell-Weisskopf theory of ionized impurity scattering are applied by taking into account the factor $\exp(\theta/2T_e)$ in the average rate of change of carrier energy due to nonpolar optical interactions. The mobility is evaluated on an electronic digital computer as a function of the carrier temperature and electric field intensity for impurity concentrations 0, 4×10^{16}, 2×10^{17}, 10^{18} and 2.5×10^{19} cm^{-3}, and also as a function of impurity concentration for low electric field intensities. The comparison of the theoretical results with the experimental data available shows a relatively good agreement.

1. Introduction

A theory of the variation of carrier mobility with electric field intensity in nonpolar semiconductors was developed some years ago by CONWELL[1,2] and BROWN[2] by including both the acoustical and nonpolar optical lattice scattering. This theory, which leads to a better agreement with experimental data for germanium than SHOCKLEY's theory[3], has been later used by SÁNCHEZ[4] for the calculation of the electron and hole mobility in n-Ge and p-Ge at a lattice temperature of 300 °K by including, in addition, the ionized impurity scattering. However, as most recently demonstrated by CONWELL[5] and in agreement with STRATTON[6], in a result quoted in Ref. [1] [Eq. (8)] the factor $\exp(\theta/2T_e)$ was omitted, and subsequently not included in the calculations of Ref. [4], thus giving rise to a small error in the mobility versus field curves obtained, without influencing the carrier temperature dependence and low-field impurity concentration dependence of the mobility computed.

The purpose of the present paper is to report the results obtained by inclusion of this factor in the previous calculations[4] and to compare the theoretical results with the experimental data available.

2. Theory of the Field Dependence of Mobility

The major premise of the present theory of hot carrier mobility is that the predominant scattering is by single phonons of acoustical and nonpolar optical lattice modes and by ionized impurities. Application of the theory is simplified by assuming spherical constant energy surfaces and parabolic bands for the carriers. With these simplifications CONWELL[1,2] and BROWN[2] obtain for the relaxation time for acoustical mode scattering:

$$\frac{1}{\tau_{ac}} = \frac{v}{4\,l_{ac}}\left(\frac{2kT}{cmv}\right)^4 \int_0^{cmv/kT} u^4 \coth u \,\mathrm{d}u\,, \qquad (1)$$

where

$$l_{ac} = \frac{\pi\hbar^4 c_l}{m^2 E_{1ac}^2 kT} \qquad (2)$$

is the mean free path for deformation potential scattering by acoustic phonons obeying equipartition, v the velocity of the carrier, k Boltzmann's constant, T the absolute lattice temperature, m the effective mass of the carrier, c the sound velocity in the crystal, c_l an average longitudinal elastic constant of the crystal, E_{1ac} the deformation potential for acoustical mode scattering and $2\pi\hbar$ Planck's constant.

With the above simplifications CONWELL[1,2] and BROWN[2] obtain further for the relaxation time for nonpolar optical mode scattering:

$$\frac{1}{\tau_{op}} = \frac{(E_{1op}/E_{1ac})^2\,\theta(2/m)^{1/2}}{2l_{ac}T[\exp(\theta/T)-1]}\left[\left(\frac{mv^2}{2}+k\theta\right)^{1/2} + \exp\left(\frac{\theta}{T}\right)\left(\frac{mv^2}{2}-k\theta\right)^{1/2}\right], \qquad (3)$$

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[2] E. M. CONWELL and A. L. BROWN, J. Phys. Chem. Solids **15**, 208 [1960].
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[5] E. M. CONWELL, High Field Transport in Semiconductors, Solid State Physics, Suppl. 9. Academic Press, New York 1967.
[6] R. STRATTON, Solid State Phys. Electron. Telecommun., Proc. Intern. Conf. Brussels, 1958, Vol. I, part I, p. 343. Academic Press, New York 1960.

where θ is the characteristic temperature of the optical phonons and E_{1op} the deformation potential for optical mode scattering. The term

$$\exp(\theta/T)(m v^2/2 - k\theta)^{1/2}$$

is present only if $m v^2/2 > k\theta$.

When the constant energy surfaces are spherical, a relaxation time exists for the ion scattering, which, conforming to the CONWELL-WEISSKOPF formulation[7], is given by

$$\frac{1}{\tau_I} = \frac{1}{2}\pi N_I^{1/3} v \frac{\ln(1+b)}{b}, \tag{4}$$

where N_I is the concentration of ionized impurities, e the electronic charge, ε the dielectric constant and

$$b = (2\pi\varepsilon m v^2/e^2 N_I^{1/3})^2. \tag{5}$$

The mobility is then obtained by averaging τ, the reciprocal of the sum of (1), (3) and (4), over the carrier distribution according to [8]:

$$\mu = \frac{e}{3m}\left\langle \frac{1}{v^2}\frac{\mathrm{d}(v^3\tau)}{\mathrm{d}v}\right\rangle. \tag{6}$$

Since it is felt that the correct carrier distribution is not known because the effect of carrier-carrier collisions has not been taken into account, a Maxwell-Boltzmann distribution is used. Combination of the τ's of (1), (3) and (4) leads to a rather complicated integral for μ as a function of the carrier temperature T_e and this is evaluated by a computer, as described in Ref. [4], by preserving the energy dependence of $\ln(1+b)$ in Eq. (4).

To determine the unknown quantities $(E_{1op}/E_{1ac})^2$ and l_{ac} at $T = 300\,^\circ\mathrm{K}$, μ is evaluated from Eq. (6) for $T_e = T$, $N_I = 0$ and for various values of these two unknown quantities, and the resulting μ versus T compared with the observed temperature dependence of the lattice mobility[9], given by

$$\mu_0 = 4.90 \times 10^7\, T^{-1.66}\ \mathrm{cm^2/Vs} \text{ for electrons}, \tag{7}$$

and

$$\mu_0 = 1.05 \times 10^9\, T^{-2.33}\ \mathrm{cm^2/Vs} \text{ for holes}. \tag{8}$$

This temperature dependence can then be well duplicated by an appropriate choice of the unknown parameters, as described in Ref. [4].

To determine how T_e varies with the electric field intensity E it is convenient to use the requirement that in the steady state, in a homogeneous semiconductor, the average power gain from the electric field equals the average power loss to the acoustic and optical modes, i.e.:

$$e\mu E^2 + \left\langle \frac{\mathrm{d}(m v^2/2)}{\mathrm{d}t}\right\rangle_{ac} + \left\langle \frac{\mathrm{d}(m v^2/2)}{\mathrm{d}t}\right\rangle_{op} = 0. \tag{9}$$

In this expression the power loss to the ionized impurities is neglected, since the mass of an impurity ion is much greater than the carrier mass.

The expressions for the average rates of loss to the acoustic and nonpolar optical modes for a Maxwell-Boltzmann distribution are[3,5,6]:

$$\left\langle \frac{\mathrm{d}(m v^2/2)}{\mathrm{d}t}\right\rangle_{ac} = -\frac{8(2kTm)^{1/2}c^2}{\pi^{1/2} l_{ac}}\left(\frac{T}{T_e}\right)^{-3/2}\left(1 - \frac{T}{T_e}\right), \tag{10}$$

$$\left\langle \frac{\mathrm{d}(m v^2/2)}{\mathrm{d}t}\right\rangle_{op} = -\frac{(E_{1op}/E_{1ac})^2 k^{3/2}\theta^3}{(2\pi m T_e)^{1/2}[\exp(\theta/T) - 1]\, T l_{ac}} \times K_1\left(\frac{\theta}{2T_e}\right)\left[\exp\left(\frac{\theta}{T} - \frac{\theta}{T_e}\right) - 1\right]\exp\left(\frac{\theta}{2T_e}\right), \tag{11}$$

where K_1 is a Bessel function of the second kind with imaginary argument[10].

3. Numerical Results for Germanium at 300 °K

The results of the computer calculation of μ/μ_0 versus T_e obtained for n-Ge and p-Ge at $T = 300\,^\circ\mathrm{K}$ by the theory just described are shown in Figs. 1 and 2. These results were obtained for $N_I = 0$, 4×10^{16}, 2×10^{17}, 10^{18} and $2.5 \times 10^{19}\ \mathrm{cm^{-3}}$ with the numerical method described in Ref. [4]. The values of the parameters used for the calculation are the following:

$\theta = 432\,^\circ\mathrm{K}$ (see Ref. [11]),
$m = 0.22\, m_0$ for electrons[1],
$m = 0.3\, m_0$ for holes[1],
$c = 5.4 \times 10^5\ \mathrm{cm/s}$ (see Ref. [3]),
$\varepsilon_r = 16$ (see Ref. [12]),
$(E_{1op}/E_{1ac})^2 = 0.4$ for electrons[4],
$(E_{1op}/E_{1ac})^2 = 4$ for holes[4],
$l_{ac} = 1.608 \times 10^{-5}$ cm for electrons at $T = 300\,^\circ\mathrm{K}$[4],
$l_{ac} = 2.355 \times 10^{-5}$ cm for holes at $T = 300\,^\circ\mathrm{K}$[4],

[7] E. M. CONWELL and V. F. WEISSKOPF, Phys. Rev. **77**, 388 [1950].
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[11] B. N. BROCKHOUSE and P. K. IYENGAR, Phys. Rev. **111**, 747 [1958].
[12] E. M. CONWELL, Proc. Inst. Radio Engrs. **40**, 1327 [1952].

where m_0 is the free electron mass and ε_r the relative dielectric constant. The values of $(E_{1op}/E_{1ac})^2$ and l_{ac} at $T = 300$ °K were chosen so that Eqs. (7) and (8) were well satisfied.

With the same values of the parameters and for $T_e = T = 300$ °K the results of the computer calculation of the electron and hole mobility μ_n and μ_p versus N_I obtained for germanium by the theory described in section 2 are shown by the curves numbered 2 in Figs. 3 and 4. These results were also obtained with the numerical method described in Ref. [4].

The results of the computer calculation of μ/μ_0 versus E obtained for n-Ge and p-Ge at $T = 300$ °K by the theory described in section 2 (including the factor $\exp(\theta/2T_e)$ in Eq. (11)) are shown in Figs. 5 and 6. These results were obtained for $N_I = 0$, 4×10^{16}, 2×10^{17}, 10^{18} and 2.5×10^{19} cm^{-3} with the same values of the parameters used for the calculation of μ/μ_0 versus T_e.

The relative errors of the calculations reported in this section are smaller than 2%. The new calculations were carried out with the aid of a Control Data 1604-A computer.

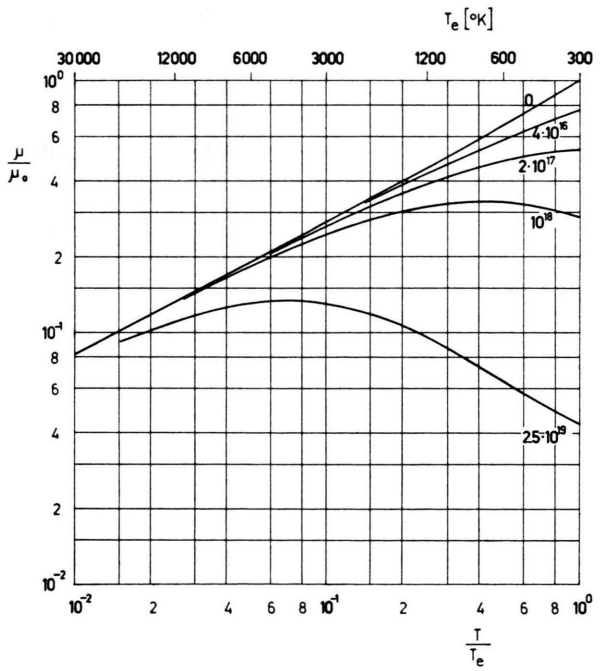

Fig. 1. Theoretical μ/μ_0 versus T/T_e and T_e for electrons in Ge at $T = 300$ °K and for five values of N_I. The numbers labeling individual curves are the N_I values in cm^{-3}.

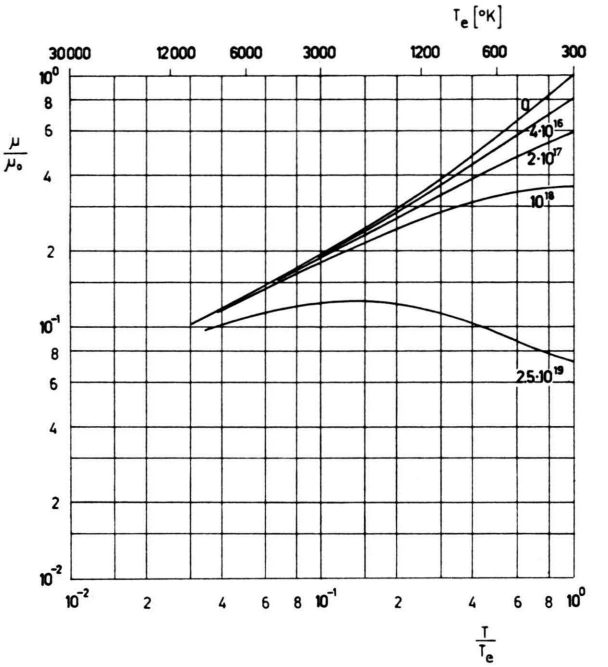

Fig. 2. Theoretical μ/μ_0 versus T/T_e and T_e for holes in Ge at $T = 300$ °K and for five values of N_I. The numbers labeling individual curves are the N_I values in cm^{-3}.

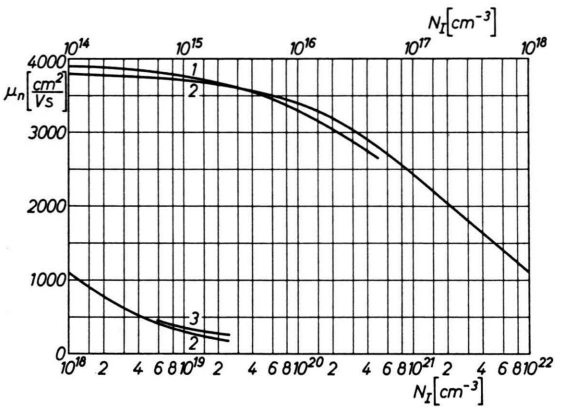

Fig. 3. Electron mobility versus impurity concentration in Ge for $T = T_e = 300$ °K. The curve 1 represents the experimental data of PRINCE[13], the curve 3 those of SPITZER, TRUMBORE and LOGAN[14] and the curve 2 the theoretical μ_n versus N_I. The scale at the top is valid for the curve 1 and the top part of the curve 2. The scale at the bottom is valid for the curve 3 and the bottom part of the curve 2.

4. Comparisons of Theory with Experiment

For the purpose of comparing the theoretical results reported in section 3 with experiment the curves for $N_I = 0$ in Figs. 5 and 6 and the experi-

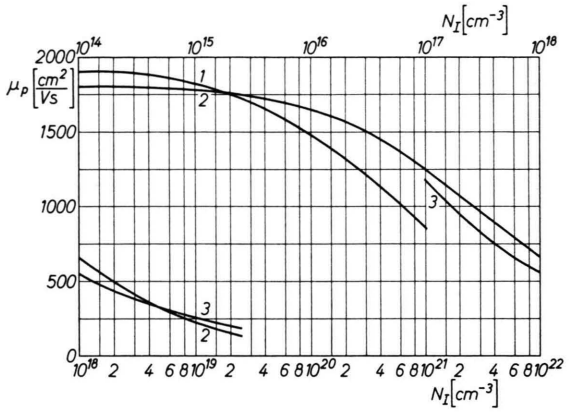

Fig. 4. Hole mobility versus impurity concentration in Ge for $T = T_e = 300\,°K$. The curve 1 represents the experimental data of PRINCE[13], the curve 3 those of TRUMBORE and TARTAGLIA[15] and the curve 2 the theoretical μ_p versus N_I. The scale at the top is valid for the curve 1 and the two top parts of the curves 2 and 3. The scale at the bottom is valid for the two bottom parts of the curves 2 and 3.

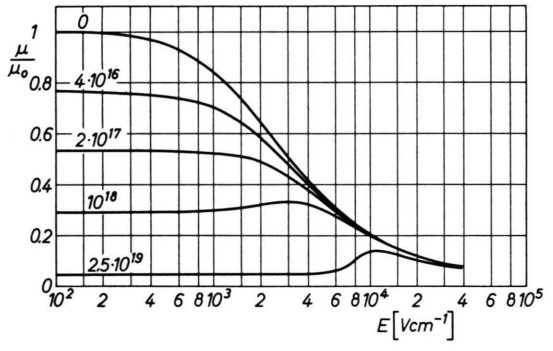

Fig. 5. Theoretical μ/μ_0 versus E for electrons in Ge at $T = 300\,°K$ and for five values of N_I. The numbers labeling individual curves are the N_I values in cm^{-3}.

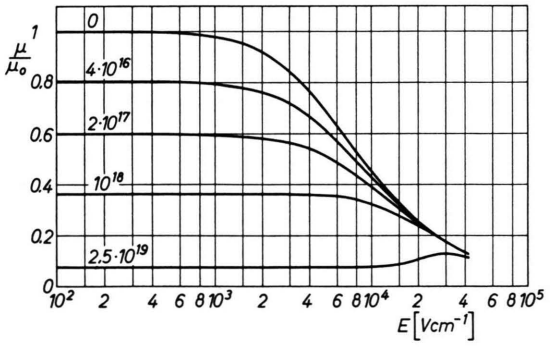

Fig. 6. Theoretical μ/μ_0 versus E for holes in Ge at $T = 300\,°K$ and for five values of N_I. The numbers labeling individual curves are the N_I values in cm^{-3}.

[13] M. B. PRINCE, Phys. Rev. **92**, 681 [1953].
[14] W. G. SPITZER, F. A. TRUMBORE, and R. A. LOGAN, J. Appl. Phys. **32**, 1822 [1961].

mental data of Ref. [1] for μ/μ_0 versus E are plotted in Fig. 7. In this figure the experimental points of GUNN, MANY and ZUCKER[1] for n-Ge at 300 °K are fitted by the curve numbered 4 and those of MANY and ZUCKER[1] for p-Ge at 300 °K by the curve numbered 2. Our corresponding theoretical curves are numbered 3 and 1 in the same figure.

The further experimental data available are shown as the curves numbered 1 and 3 in Figs. 3 and 4, giving the low-field majority carrier mobility μ_n and μ_p as a function of N_I in germanium at 300 °K. In these figures the curves numbered 1, after PRINCE[13], were obtained from drift mobility measurements. The experimental points of SPITZER, TRUMBORE and LOGAN[14] for low-field μ_n in As-doped Ge are fitted by the curve numbered 3 in Fig. 3 and those of TRUMBORE and TARTAGLIA[15] for low-field μ_p by the curve numbered 3 in Fig. 4. For the purpose of comparing these experimental data with the theory in section 2 our theoretical curves, numbered 2, are plotted in the same Figs. 3 and 4.

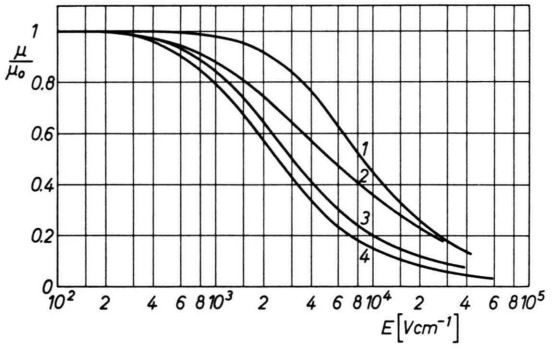

Fig. 7. μ/μ_0 versus E for $N_I = 0$ in Ge at $T = 300\,°K$. The curve 1 represents the theoretical μ/μ_0 versus E for holes and the curve 3 that for electrons. The curve 2 represents the experimental data of MANY and ZUCKER[1] for holes and the curve 4 those of GUNN, MANY and ZUCKER[1] for electrons.

5. Discussion of Theoretical Results

The curve numbered 3 in Fig. 7 differs from the corresponding theoretical curve of Ref. [1], for all the values of E, by less than 2%, the upper limit of the relative errors of our calculations. The relatively good agreement of theory and experiment must be considered, however, at the highest

[15] F. A. TRUMBORE and A. A. TARTAGLIA, J. Appl. Phys. **29**, 1511 [1958].

fields as largely fortuitous. As pointed out by CONWELL[5], this theoretical curve cannot be considered to be meaningful beyond a field of 5×10^3 V/cm because the scattering to the $\langle 100 \rangle$ minima, 0.2 eV above the band edge, is not included in the theory. A further source of error is that the use of the simple model makes it impossible to take into account the effect of scattering among the $\langle 111 \rangle$ valleys. The choice of a Maxwell-Boltzmann distribution, however, has been shown, in the case of n-Ge, to be well justified for most of the range of fields involved[16].

In the case of p-Ge the curve numbered 1 in Fig. 7 differs, in general, from the corresponding theoretical curve of Ref. [1] also by less than 2%. Only around 10^4 V/cm our theoretical curve agrees about 4% better with the experimental data. The agreement of theory and experiment is not as good as that for n-Ge. One obvious shortcoming of the calculation for p-Ge is, as remarked by CONWELL[5], the use of the simple model of the band structure. The light holes, having a mobility many times μ_0, would start to heat up at much lower fields than a hole with average mobility μ_0 and, through the collisions with the heavy holes and interband transitions, also produce more heating of the heavy holes than expected at a given field. The most important reason for the discrepancy between theory and experiment in the case of p-Ge is, however, the assumption of the Maxwell-Boltzmann distribution, which, for $mv^2/2 \lesssim k\theta$, is incompatible with the very strong coupling to the optical modes[17,18].

In the present calculations the Conwell-Weisskopf theory of impurity scattering was used instead of the BROOKS-HERRING theory[19,20], which takes account of the screening of the ion by the surrounding carrier cloud. This effect does not influence, however, the majority carrier mobility in homogeneously doped samples, with only donors or acceptors, because the majority carriers are then uniformly spread about around all the ions so that both the majority carriers and ions are effectively shielded and, therefore, during the scattering of a majority carrier by an ion the carrier sees only the field of this ion. For this reason, the comparison of the theoretical curves in Figs. 3 and 4 is made with the experimental data for majority carrier mobility. The agreement of theory and experiment is similar to that shown in Fig. 7. Some reasons for the discrepancy are the possible measurement errors in the determination of the experimental curves. A further reason for the discrepancy in Figs. 3 and 4 may be the approximation made by PRINCE[13], to obtain the curves numbered 1, viz. that the Conwell-Weisskopf formula holds for electron-hole scattering.

As shown in Figs. 1—7, our calculations were carried out for impurity concentrations up to 2.5×10^{19} cm^{-3}, which is approximately the degenerancy concentration at room temperature with the effective mass taken as the mass of the free electron[21].

In conclusion, it should be remarked that, despite the simplifications in treating band structure and distribution function, the calculations in this paper should serve to give a good idea of the variation of carrier mobility with electric field and impurity concentration in homogeneous nonpolar semiconductors. They also tend to give a good picture of the variation of carrier mobility with carrier temperature and impurity concentration in inhomogeneous nonpolar semiconductors. This holds especially for some semiconductor devices, for which "exact" calculations of the electric field distribution, the electron and hole concentration distribution and the electron and hole current density distribution are much more difficult to carry out, even when the Maxwell-Boltzmann distribution and the simple model of the band structure are used[22,23].

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